Class Exercise 1.

1. Prove that $\sqrt{2}|z| \geq |\operatorname{Re} z| + |\operatorname{Im} z|$. Deduce the maximum value assumed by $\cos\theta + \sin\theta$.

2. In each case below, sketch the set of points determined by the given condition:

   (a) $|z+1-i| = 1$; (b) $|z-i| \leq 3$; (c) $\operatorname{Re}(\bar{z}+i) = 2$; (d) $|z-i| = |z+i|$.

3. (a) Given that $z_1 z_2 \neq 0$, use the polar form to prove that $\operatorname{Re}(z_1.\bar{z}_2) = |z_1|.|z_2|$ iff $\theta_1 - \theta_2 = 2n\pi$ $(n \in Z, \theta_1 = \arg z_1, \theta_2 = \arg z_2)$.

   (b) Deduce that $|z_1 + z_2| = |z_1| + |z_2|$ iff $\theta_1 - \theta_2 = 2n\pi$ $(n = 0, \pm 1, \ldots)$ where $\theta_1 = \arg z_1, \theta_2 = \arg z_2$. Verify this statement geometrically.

4. In each case below, find all the roots and exhibit them geometrically:

   (a) $(5i)^{\frac{1}{2}}$; (b) $i^{\frac{1}{4}}$; (c) $(-1)^{\frac{1}{4}}$; (d) $4^{\frac{1}{5}}$.

5. Find the four roots of the equation $z^4 + 4 = 0$, and use them to factor $z^4 + 4$ into quadratic factors with real coefficients.