Class Exercise 2.

1. Assuming the sufficiency theorem for differentiablity, and that $f'(z) = u_x + iv_x$, show that $f'(z)$ and its derivative $f''(z)$ exist everywhere, and find $f'(z)$ and $f''(z)$ when

   (a) $f(z) = 2z + i$; (b) $f(z) = e^{-x}e^{-iy}$; (c) $f(z) = z^3$.

2. If $f(z) = x^3 - i(y-1)^3$, then $u_x(x,y) + iv_x(x,y) = 3x^2$. Why is it true that $f'(z) = 3x^2$ only at the point $z = i$?

3. Show that u is harmonic in some domain and find a harmonic conjugate v when

   (a) $u(x,y) = 2x(1-y)$; (b) $u(x,y) = 2x - x^3 + 3xy^2$;
   (c) $u(x,y) = \sinh x.\sin y$.

4. Show that

   (a) $\exp(2 \pm 5\pi i) = -e^2$; (b) $\exp\left(\frac{2+\pi i}{4}\right) = \sqrt{e}(1+i)/\sqrt{2}$;
   (c) $\exp(z - \pi i) = -\exp z$.