

Class Exercise 3.

1. Find the image of the quadrant $x > 1, y < 0$ under the transformation $w = 1/z$. Point out the images of the boundaries of the quadrant.
2. Express the mapping $w = f(z) = (z + 1)/(z - 1)$ as a combination of simpler mappings. Hence find the image of the half-plane $x > 0$ under the mapping $w = f(z)$.
3. Show that $\sin z = \sin x \cosh y + i \cos x \sinh y$.
[Harder!] By considering the images of vertical lines $x = c$ ($-\pi/2 \leq c \leq \pi/2$), show that $w = \sin z$ is a 1 - 1 mapping of the strip $-\pi/2 \leq x \leq \pi/2, y \geq 0$ onto the upper half of the w -plane. Describe corresponding portions of the boundaries.
4. Find the image of the semi-infinite strip $x \geq 0, 0 \leq y \leq \pi$ under the transformation $w = \exp z$, and describe corresponding portions of the boundaries.