Class Exercise 4.

1. Let the simple closed contour C be the circle $|z| = 1$. Giving reasons, find $\int_C f(z)dz$ for each of the following functions.

   (a) $f(z) = ze^{-z}$; (b) $f(z) = 1/(z^2 + 2z + 2)$; (c) $f(z) =$ sech z;
   (d) $f(z) = \tan z$; (e) $f(z) =$ Log $(z + 2)$.

2. Show that if $z_1 \neq 0$, $z_2 \neq 0$ and $z_1 \neq z_2$, then $\int_{z_1}^{z_2} (1/z^2)dz = 1/z_1 - 1/z_2$. whenever the path of integration is interior to a simply closed domain which does not contain the origin. Show how it follows that, for any simple closed contour C where the origin is either an interior or exterior point, $\int_C (1/z^2)dz = 0$.

3. Let C denote the boundary of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$, where C is described in the positive (anti-clockwise) sense. Evaluate each of these integrals:

   (a) $\displaystyle\int_C \frac{e^{-z}}{z - \pi i/2}dz$; (b) $\displaystyle\int_C \frac{\cos z}{z(z^2 + 8)}dz$; (c) $\displaystyle\int_C \frac{z}{2z + 1}dz$.

4. Let C be the unit circle $z =$ exp $(i\theta)$ described from $\theta = -\pi$ to $\theta = \pi$. First show that, for any real constant a, $\int_C e^{az}/z dz = 2\pi i$.

   Then write the integral in terms of θ to derive the formula

$$\int_0^{\pi} e^{a\cos\theta} \cos(a \sin\theta)d\theta = \pi.$$