Class Exercise 5.

1. By considering the remainder $R_N(z)$, show that

$$\sum_{n=1}^{\infty} z^n = \frac{z}{(1-z)}$$

where z is any complex number such that $|z| < 1$.

2. Show that if $\sum_{n=1}^{\infty} z_n = S$ then $\sum_{n=1}^{\infty} \bar{z_n} = \bar{S}$.

3. Show that if z is the limit of the sequence z_n $(n = 1, 2, 3, \ldots)$ and if $|z_n| \leq M$ for all n, then $|z| \leq M$.

4. Show that $e^z = e + e\sum_{n=1}^{\infty}(z-1)^n/n!$ when $|z| < \infty$.

5. Prove that when $0 < |z| < 4$,

$$\frac{1}{4z - z^2} = \sum_{n=0}^{\infty} \frac{z^{n-1}}{4^{n+1}}.$$