Class Exercise 6.

1. Give two Laurent series expansions in powers of z for the function $f(z) = \dfrac{1}{z^2(1+z)}$, and specify the regions in which these expansions are valid.

2. In each case below write the principal part of the function at its isolated singular point. Determine if that point is a pole, an essential singular point, or a removable singular point of the given function.

   (a) $ze^{\frac{1}{z}}$; (b) $\dfrac{z^2}{1-z}$; (c) $\dfrac{\sin z}{z}$; (d) $\dfrac{\cos z}{z}$.

3. Show that all the singular points of each of the following functions are poles. Determine the order m of each pole and find the corresponding residue K.

   (a) $\dfrac{z+1}{z^2+2z}$; (b) $\dfrac{1-\ \exp\,(2z)}{z^4}$; (c) $\dfrac{\exp\,(2z)}{(z+1)^2}$; (d) $\dfrac{\exp\, z}{z^2+\pi^2}$.

4. Find the value of the integral $\displaystyle\int \frac{1}{z^3(z+4)}dz$ taken counterclockwise around circle (a) $|z| = 2$, (b) $|z+2| = 3$.