Tutorial 1.

1. In each case below sketch the set and determine whether it is a domain (open and connected):

   (a) $|z+2+i| \leq 1$; (b) $|2z-3| \geq 4$; (c) $\text{Im } z < 1$; (d) $|\text{ Im } z| > 1$;

   (e) $|z| > 0,\ 0 \leq \text{ arg } z \leq \pi/3$; (f) $|z+4| \geq |z|$; (g) $0 < |z - z_0| < \delta$, where z_0 is a fixed point, and δ is a positive real number.

2. (a) Which sets in Q1 are neither open nor closed?

   (b) Which sets in Q1 are bounded?

3. (a) If u, v are complex numbers, prove that $|u|.|v| = |uv|$.

   (b) If $\lim_{z\to z_0} f(z) = w$, show that $\lim_{z\to z_0} |f(z)| = |w|$.

   [Note that this means that $\lim_{z\to z_0} |f(z)| = |\lim_{z\to z_0} f(z)|$.]

   (c) Carefully show that the function $f(z) = |z|^2$ is continuous in the entire complex plane.

4. (a) Show that the function $f(z) = \bar{z}$ is nowhere differentiable.

   (b) Determine whether the function $f(z) = \text{ Im } z$ has a derivative at any point.

5. Investigate the way in which complex numbers might be used to represent simple mappings in the plane. How would you represent (a) a translation (b) a rotation about the origin (c) a reflection in the x-axis?

   What about a rotation about a point other than the origin? A reflection in some general line through the origin? A glide reflection?