

Tutorial 2.

1. (a) Write $|\exp(3z + i)|$ in terms of x and y .
(b) Write $|\exp(iz^2)|$ in terms of x and y .
(c) Show that $|\exp(2z + i) + \exp(iz^2)| \leq e^{2x} + e^{-2xy}$.
2. (a) Show that $\operatorname{Log}(\sqrt{5} + 2) = -\operatorname{Log}(\sqrt{5} - 2)$.
(b) Find all the roots of $\cos z = 2$.
3. Show that if $\operatorname{Re} z_1 > 0$ and $\operatorname{Re} z_2 > 0$ then $\operatorname{Log}(z_1 z_2) = \operatorname{Log} z_1 + \operatorname{Log} z_2$.
4. Show that
 - (a) if $\log z = \operatorname{Log} r + i\theta$ ($r > 0, \pi/4 < \theta < 9\pi/4$), then $\log(i^2) = 2 \log i$.
 - (b) if $\log z = \operatorname{Log} r + i\theta$ ($r > 0, 3\pi/4 < \theta < 11\pi/4$), then $\log i^2 \neq 2 \log i$.
5. Find the principal value of
 - (a) i^{3i} ;
 - (b) $[\frac{e}{2}(-1 + i\sqrt{3})]^{3\pi i}$.