Tutorial 3.

Note: Do not use Cauchy's Theorem to solve these problems.

1. Evaluate
   (a) $\int_0^{\pi/4} e^{it} dt$;
   (b) $\int_0^{2\pi} e^{imt} e^{-int} dt$ (m, n integers).
2. Let $f(t) = u(t) + iv(t)$ be a piecewise continuous complex-valued function of a real variable t defined on an interval $a \leq t \leq b$. Show that if $F(t) = U(t) + iV(t)$ is a function such that $F'(t) = f(t)$, then

$$\int_a^b f(t)dt = F(b) - F(a).$$

For each contour C in Questions 3, 4 find the value of $\int_C f(z)dz$.
Is C a contour? Is f piecewise continuous?

3. $f(z) = (z - 2)/z$, and
   (a) the semicircle $z = 2e^{i\theta}(0 \leq \theta \leq \pi)$;
   (b) the semicircle $z = 2e^{i\theta}(\pi \leq \theta \leq 2\pi)$;
   (c) the circle $z = 2e^{i\theta}(0 \leq \theta \leq 2\pi)$.
4. $f(z) = e^z$ and C is the arc from $z = \pi i$ to $z = 1$ consisting of (a) the line segment joining these points; (b) the portion of the coordinate axes joining these points.