Tutorial 4.

1. Let C be the circle $|z| = 3$ described in the positive sense.

   Set $g(z_0) = \displaystyle\int_C \frac{2z^2 - z - 2}{z - z_0} dz \ (|z_0| \neq 3)$.

   (a) Find $g(2)$. (b) What is the value of $g(z_0)$ when $|z_0| > 3$?

2. Let C be a simple closed contour described in the positive sense, and write

   $g(z_0) = \displaystyle\int \frac{z^3 + 2z}{(z - z_0)^3} dz.$

   Find $g(z_0)$ (a) when z_0 is inside C and (b) when z_0 is outside C.

3. Find the value of the integral of $g(z)$ around the simple closed contour $|z - i| = 2$ in the positive sense when

   (a) $g(z) = 1/(z^2 + 4)$; (b) $g(z) = 1/(z^2 + 4)^2$.

4. Let function f be continuous in a closed bounded region R and analytic and not constant throughout the interior of R. Assuming that $f(z) \neq 0$ anywhere in R, consider the function $1/f(z)$ to prove that $|f(z)|$ has a minimum value N somewhere in R, and that $|f(z)| > N$ for each point z in the interior. (This is a Minimal Modulus Theorem.)

5. Give an example to show that in Q4 the condition $f(z) \neq 0$ anywhere in R is necessary to prove the result. That is, show that $|f(z)|$ can reach its minimum value at an interior point when that minimum value is zero.