Tutorial 5.

1. (a) Use the Taylor Theorem to expand $\sin z$ into a Taylor series about the point $z = \pi/2$.

   (b) In retrospect, was there a smart way of doing this?

2. (a) Expand the function $1/z$ into a series of powers of $z - 1$.

   (b) Use (a) to obtain by differentiation the expansion of $1/z^2$ as a series of powers of $z - 1$.

   Give the region of validity in each case.

3. Obtain the expansion of the function $(z + 1)/(z - 1)$ by

   (a) its Maclaurin series, and give the region of validity for the representation;

   (b) its Laurent expansion for the domain $|z| > 1$.

4. Expand the function $(z - 1)/z^2$ into

   (a) its Taylor series in powers of $z - 1$, and give its region of validity;

   (b) its Laurent series for the domain $|z - 1| > 1$.

   [Hint: vary the ideas used in Q2.]

5. Write the Laurent series expansion of the function $1/(z - k)$for the domain $|z| > |k|$, where k is real and $-1 < k < 1$. Then write $z = e^{i\theta}$ to obtain the formulae

$$\sum k^n \cos n\theta = \frac{k\cos\theta - k^2}{1 - 2k\cos\theta + k^2}, \quad \sum k^n \sin n\theta = \frac{k\sin\theta}{1 - 2k\cos\theta + k^2}.$$